Course of Advanced Automation and Control

Exam for the students of the a.y. 2016/2017

September 08, 2017

Surname ______________________ Name ______________________

Part I - Optimization & Graphs (Prof. D.M. Raimondo)

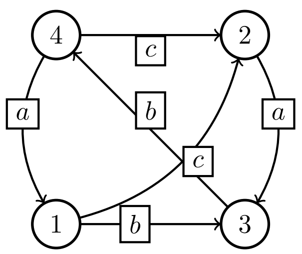
1. The Grecchi S.p.A. produces $750ml$ glass wine bottles that sells at the price of $0.25€/unit$. The production costs are $0.15€/unit$. The company sells on average 3 million bottles per year. The owner of the company is considering the purchase of a new technology for the price of $200K€$ which would lead, by reducing the energy consumption, to a lowering of the production costs by $0.03€/unit$. The company can choose among two strategies:

   - keep the same selling price and benefit directly of the production cost reduction
   - reduce the selling price by $0.03€/unit$.

   The second solution would lead to an increase of requests from 3 to 4 million bottles per year (the price reduction is possible only if the new tech is acquired). However, if the production exceeds the 3.5 million bottles per year, it is necessary to expand the production site. This comes at the fixed cost of $500K€$. The company can therefore chose whether to produce at most 3.5 million bottles or expand the site. The objective of Grecchi S.p.A. is to evaluate if the investments are worthy. In particular, the company wants to maximize the profits over the next 10 years. Please formulate the problem above as a mixed integer linear program to support the decision-making process of Mr. Grecchi.

   Important note: while formulating the problem above, you will obtain bilinear terms like $prod * \delta_i$ with $prod$ indicating the bottle production and δ_i a possible binary variable. In order the problem to be an MILP, such terms need to disappear from the problem and be replaced by new variables y_i subject to the following constraints: $y_i \leq M\delta_i$, $y_i \geq m\delta_i$, $y_i \leq prod - m(1-\delta_i)$, $y_i \geq prod - M(1-\delta_i)$, with $M = max(prod)$ and $m = min(prod)$.

2. Consider the automaton in the figure ($C = \{a, b, c\}$ is the set of control values and $S = \{1, 2, 3, 4\}$ is the set of state values) with the intermediate cost $g(x, u)$ and the terminal cost $g_3(x)$ given below



$g(x,u)$	a	b	c
1	-	3	2
2	3	-	-
3	-	4	-
4	1	-	2

$$g_3(x) = \begin{cases} 4 & \text{if } x = 1 \\ 3 & \text{if } x = 2 \\ 1 & \text{if } x = 3 \\ 2 & \text{if } x = 4 \end{cases}$$

   2.1 Solve the optimal control problem

$$J(x_0) = \min_{u_0,u_1,u_2} g_3(x_3) + \sum_{k=0}^{2} g(x_k, u_k)$$

   using dynamic programming.

   2.2 Compute an optimal control sequence for $x_0 = 2$ and compute the optimal cost value.

3. Please solve the following MILP problem using the branch and bound algorithm

$$\begin{array}{lll} \max_{x_1,\delta_1,\delta_2} & -0.2x_1 + 2\delta_1 + 4\delta_2 & \\ & \delta_2 + \delta_1 & \leq 1.7 \\ & \delta_1, \delta_2 \in \{0, 1\} & \\ & x_1 \geq 0 & \end{array}$$